

### 3.3 Magnetic anomalies

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#### The characteristics of magnetic anomalies

The objective of magnetic surveys is to map the subsurface distribution of magnetization and from this infer the susceptibility and hence the magnetic mineral content of the rocks. The magnetization is detected by measuring the variations or anomalies in the Earth's field caused by the fields produced by the subsurface magnetizations.

The equation for the field of a dipole is given by equation 3.1.1. We have seen that magnetized matter is made up of a sum or integral of the

atomic or domain dipoles so the field of magnetized matter in a volume  $V$  is given by the integral of 3.1.1 over  $V$ :

$$\vec{B} = \frac{\mu_0}{4\pi} \int_V \nabla \left( \vec{M} \cdot \nabla \left( \frac{1}{r} \right) \right) dV, \quad (3.3.1)$$

where the elementary dipole moment has been replaced by the dipole moment per unit volume.

The problem with this deceptively simple formula is that the magnetization,  $\vec{M}$ , depends on the field via  $\vec{M} = \chi \vec{H}$ , and  $\vec{H}$  is not known inside the body. We have an **integral equation** with the unknown inside and outside the integral. This is known as a Fredholm integral equation and its solution can usually only be obtained numerically. The general nature of the solution can be seen by looking at the solution for bodies of simple shape. For some bodies of revolution the problem can be solved analytically as a **boundary value problem**.

The sphere is one such model and it is particularly useful because its anomaly can be used as a first order approximation for any subsurface body of compact shape. The complete solution for the boundary value problem of a sphere of radius  $R$ , susceptibility  $\chi_2$ , in a uniform inducing field  $B_0$  can be found in Level 2. For this analysis it is important to note that the magnetization is **uniform**, and is in the direction of the inducing field. Further, the field outside is that of a **dipole** with a moment given by:

$$\text{Dipole moment of sphere, } \vec{m} = H \frac{\chi_2}{\left(1 + \frac{\chi_2}{3}\right)} \times \text{Volume}$$

The moment is just the magnetization one would have calculated for the material of susceptibility  $\chi_2$ ,  $M = \chi_2 H$ , times the volume, but reduced by the **demagnetization factor** of  $1 / (1 + \frac{\chi_2}{3})$ .

An interesting practical consequence can be seen in this result. If  $\chi_2$  is less than 0.1 (S.I.) then the demagnetizing factor is negligible. If  $\chi_2$  is 1.0 the factor is only 0.75. We will find below that for most rocks  $\chi_2$  is less than 0.1 and so we don't have to worry about the effect most of the time.

With this analytic result we can return to the general integral expression, Equation 3.3.1, and simply insert  $H_0\chi$  for  $M$ . This is an approximation, called the Born approximation, and it is valid whenever  $\chi_{S.I.}$  is less than 0.1. Equation 3.3.1 is used in all the calculations of the anomalous fields for subsurface bodies that are presented in texts and papers on magnetic modeling. One must be careful in using these results to interpret the anomalies from high susceptibility bodies such as iron ore deposits or metal objects.

This approximation makes the calculation of magnetic anomalies similar to those of gravity. There is a direct mathematical relationship between gravity and magnetic anomalies for the same body known as Poisson's relationship. If the magnetization is constant throughout the volume and has direction  $\mathbf{k}$  and the density is also constant then the vector magnetic field anomaly is related to the vector  $g$  anomaly by:

$$\bar{H} = \frac{M}{G\rho} \frac{\partial}{\partial k} \bar{g}$$

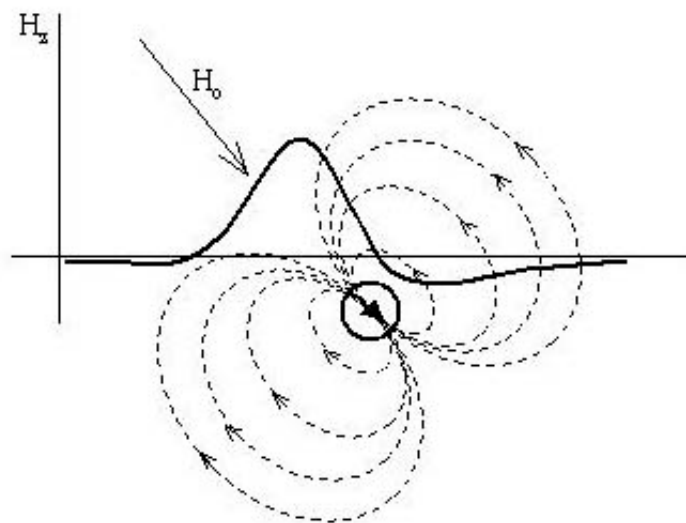
This is an extremely useful relationship because it means that you only need to calculate the gravity anomaly of a body and the magnetic anomaly can be found simply by taking the derivative of vector  $g$  in the magnetization direction.



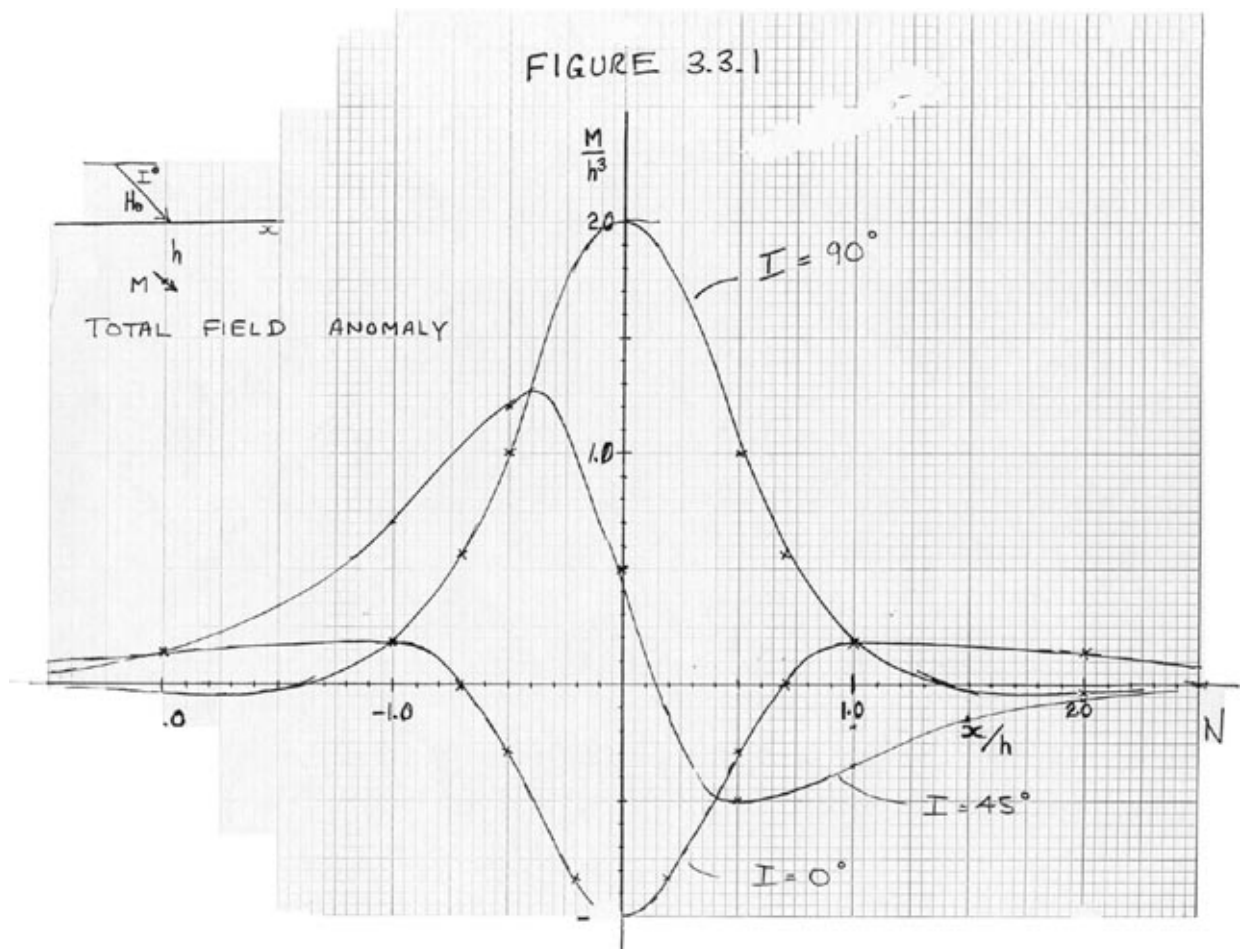
### Anomalies of simple shapes

#### The sphere (dipole)

Qualitatively the anomalies of most confined bodies can be estimated by drawing the field lines emanating from the body. The sketch below shows the anomaly in the vertical component of  $H$ ,  $H_z$ , for a body magnetized in the Earth's field direction. Usually only the anomalous field component is presented because survey results are usually referenced to a base station away from the region of interest which is defined as the zero level for the survey.



In most surveys the proton magnetometer is used and it effectively measures the anomaly in the Earth's field direction - this is called the total field anomaly. The total field anomalies for an induced dipole moment  $M$  for three inclinations  $I$  are plotted quantitatively in Figure 3.3.1. The amplitudes are normalized by  $M/h^3$  and the horizontal scale is normalized by  $h$  the depth of the dipole (sphere). The anomalies at the pole and the equator are symmetrical and the depth is about twice the half width. At an inclination of  $45^\circ$  N the anomaly is asymmetrical with a **low to the north of the body**. The depth at these mid-latitudes is roughly estimated as the horizontal distance between the peaks of the anomaly.





### The half space

$H_z$  is the only anomalous component of magnetic field from a uniformly magnetized half space. The anomalous field is:

$$H_z = 2\pi M \sin I$$

Note that the anomaly is independent of the height of the measurement point. This is not a very practical result because such half spaces don't exist, but the result is useful to derive the anomaly of a layer and to show asymptotes for the anomaly of a vertical fault.



### The layer

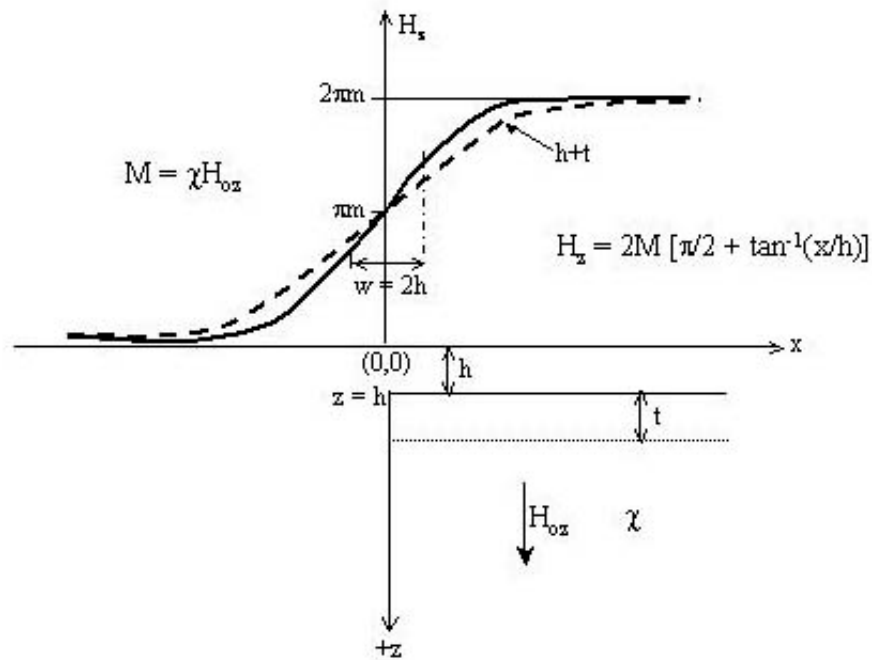
The anomalous field of a layer of thickness  $h$  is the superposition (subtraction) of two half spaces a distance  $h$  apart vertically. It is consequently zero.



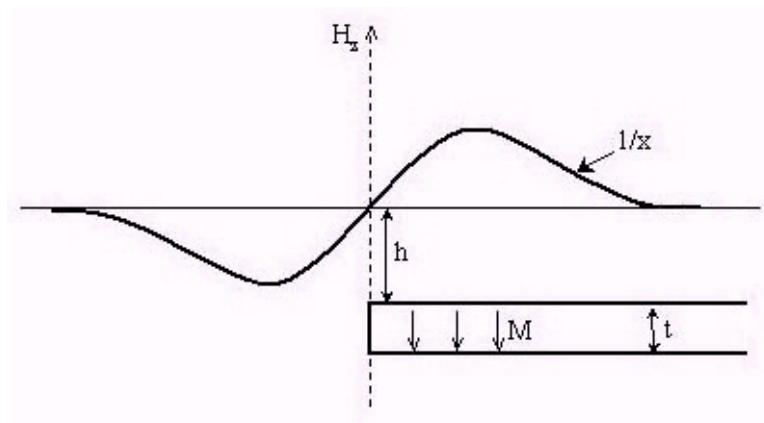
### The quarter space

The vertical contact between two media of different susceptibilities is the same as a quarter space of the difference in susceptibility. For arbitrary magnetization this is a difficult problem and requires a numerical solution.

For a vertical inducing field the solution is simpler and is given in the sketch below.



The anomaly for a truncated layer of thickness  $t$  may be obtained by subtracting the anomaly from the quarter space at depth  $h + t$  (dashed line) from that at depth  $h$  (solid line). Schematically the anomaly looks like:





### The magnetic pole

While there is no such thing as a magnetic pole, a long thin rod magnetized in the direction of its length has the basic response of a pole near one of its ends. The response is actually that of a very long dipole and the response of the far end is negligible. The vertical pipe is a practical model. The magnetization along the pipe is stronger than that transverse to the pipe so in northern latitudes the dominant effect is from the vertical magnetization which yields a pole - like response. The vertical field anomaly from a pole is symmetric and the depth is approximately 1.3 times the half width. The total field anomaly is not symmetric as the following sketch indicates.



### A line of poles

A line of poles is a reasonable approximation for the anomaly of a vertical or subvertical thin sheet. In a section perpendicular to the strike of the sheet (a dip section) the anomaly is similar in shape to that of the N - S profile over a pole.